Mechanizing the Metatheory of Sledgehammer

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Overview

- Sledgehammer
- Sledgehammer's Metatheory
- Mechanization

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- Sledgehammer's Metatheory
- Formal Verification

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Sledgehammer's Pipeline

φ

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$$(\bigwedge F) \stackrel{?}{\Longrightarrow} \varphi$$

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- If one succeeds, with $E \subseteq F$

Sledgehammer's Pipeline

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- Why reprove? Isabelle trusts no one but herself

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- Sledgehammer's Metatheory
- Formal Verification

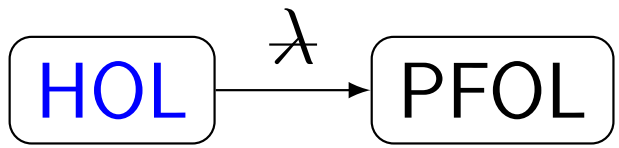
Sledgehammer's Metatheory

Higher Order Logic

HOL

Sledgehammer's Metatheory

Polymorphic First Order Logic

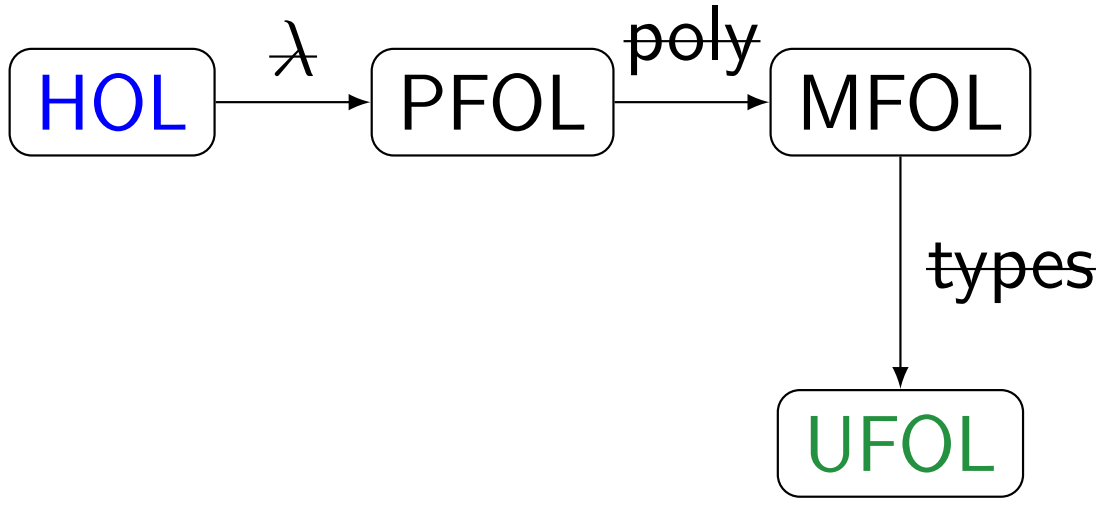


Sledgehammer's Metatheory

Many-Typed (Many-Sorted) First Order Logic

Sledgehammer's Metatheory

Untyped First Order Logic



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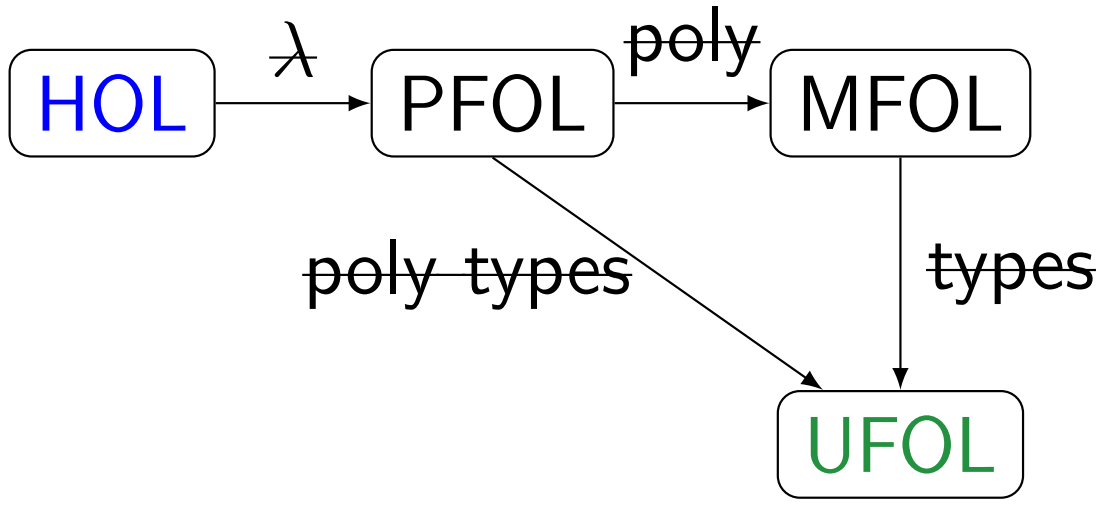

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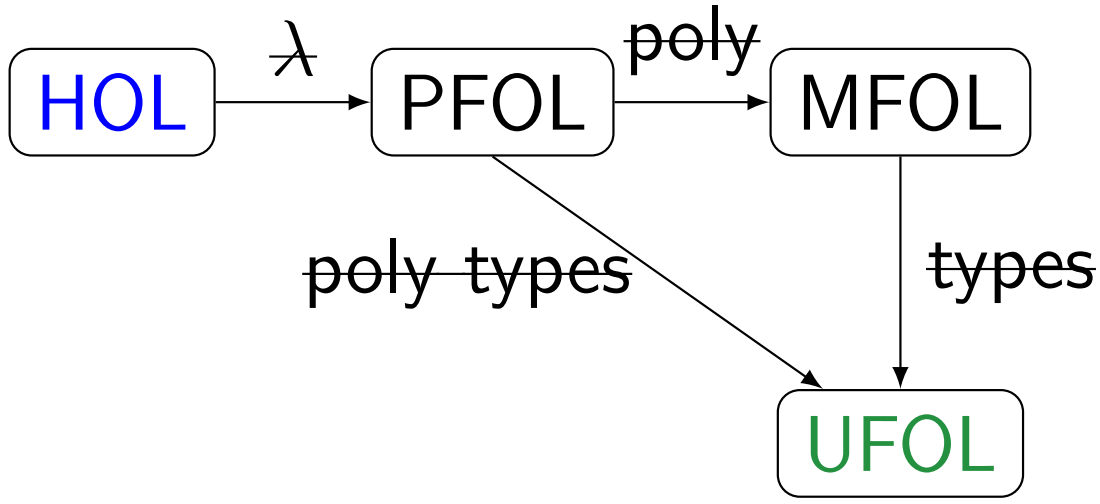
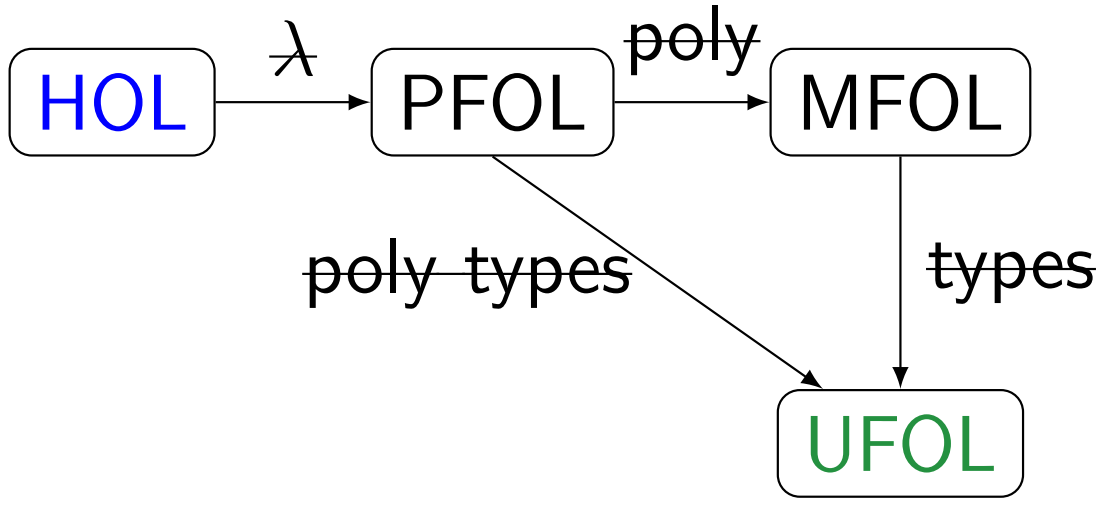


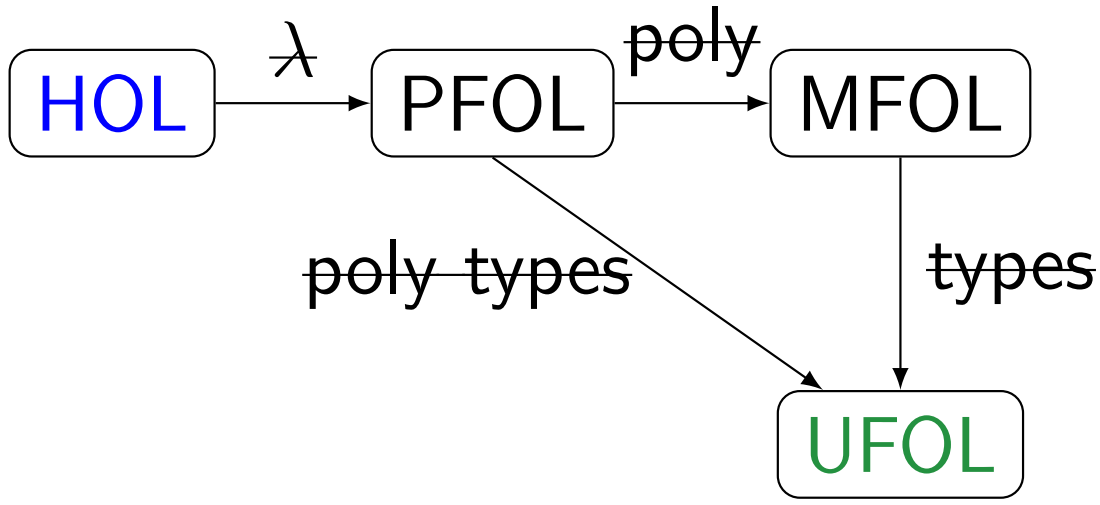
Are the encodings

Sledgehammer's Metatheory



Are the encodings sound?

Sledgehammer's Metatheory



Are the encodings sound? complete?

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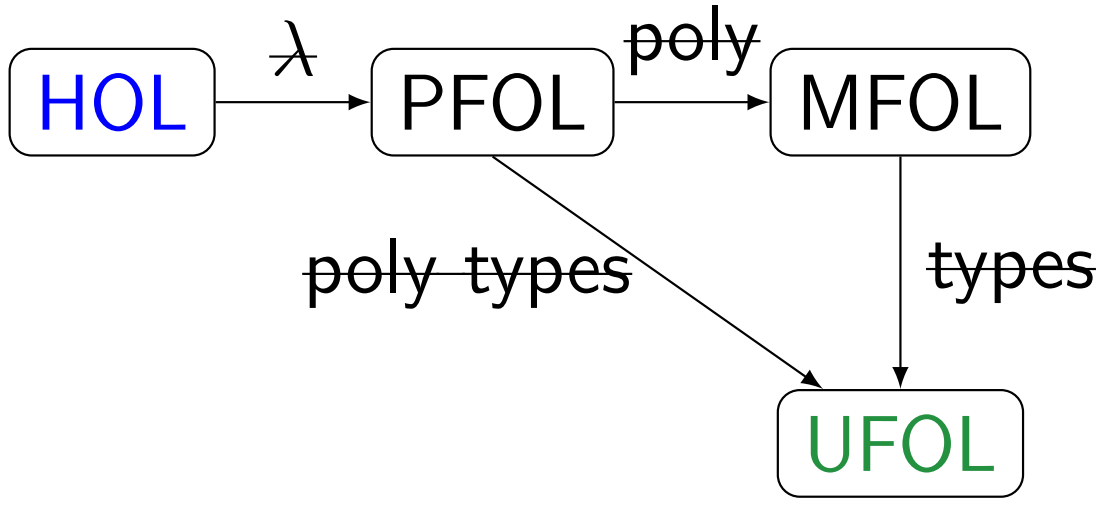

Are the encodings sound? complete? efficient?

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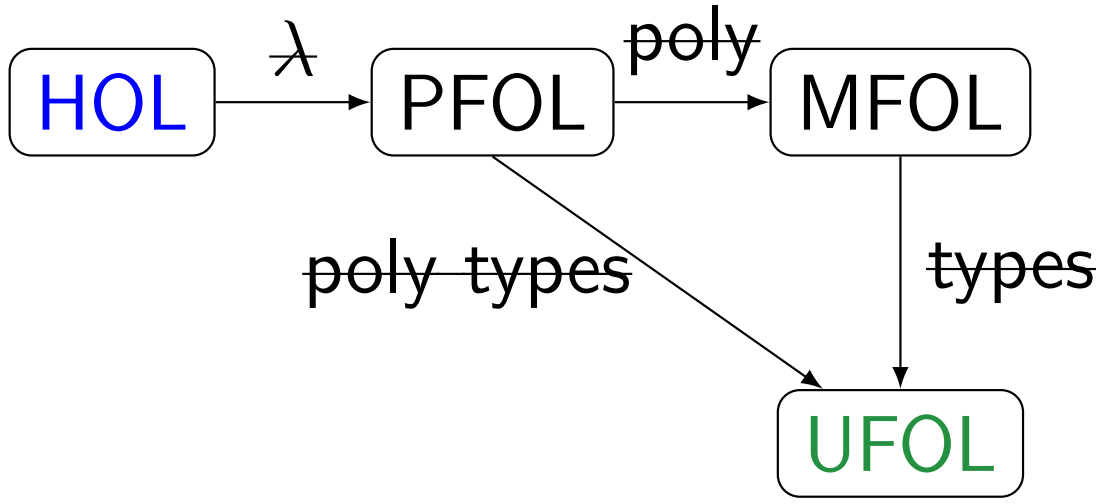

Are the encodings sound? complete? efficient?
All these are important practical questions!

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It's all for the user ☺

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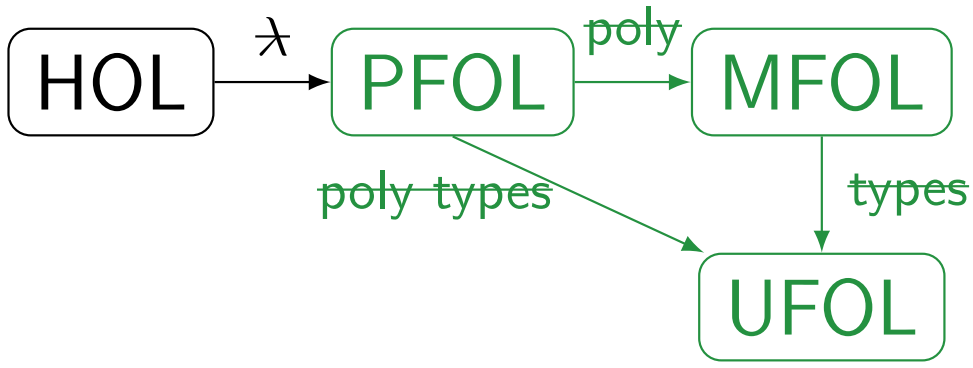
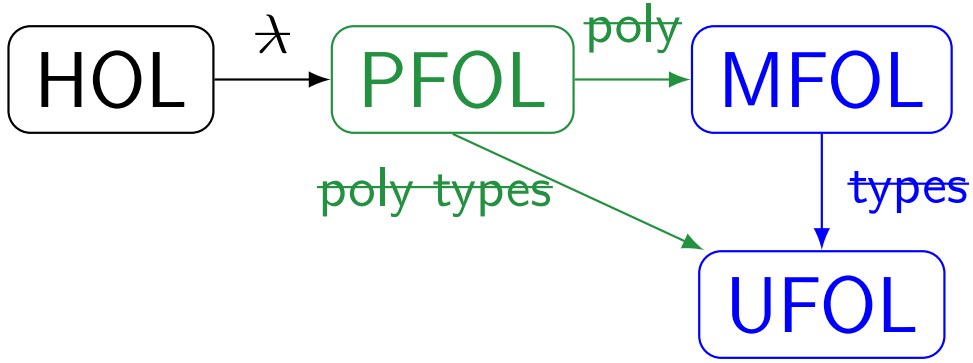
Formal Verification



Blanchette et al., TACAS 2013:
Encoding Monomorphic and Polymorphic Types
(Soundly, Completely, and Efficiently)
A variety of encodings

- based on tags or guards
- full, lightweight, featherweight
- factor in infiniteness knowledge

Formal Verification



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FroCoS 2013: Formal verification by Isabelle herself

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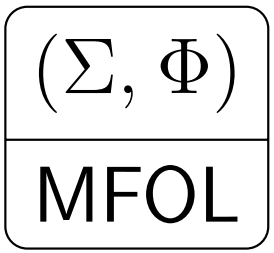
  - Codatatypes

Encodings

Σ: MFOL signature
Φ: set of Σ-formulas



Encodings

$\Sigma^{\#}$: Σ with sorts erased and func/pred added

$$\Phi^{\#} = \mathsf{Ax}_{\Phi} \cup \{\varphi^{\#} \mid \varphi \in \Phi\}$$

(Σ, Φ)
MFOL

$(\Sigma^{\#}, \Phi^{\#})$
UFOL

Encodings

Soundness

$$M \models \Phi \Longrightarrow M^{\#} \models \Phi^{\#}$$

Encodings

Completeness

$$N^{\$} \models \Phi \Longleftarrow N \models \Phi^{\#}$$

Encodings

Example

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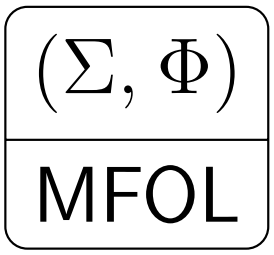
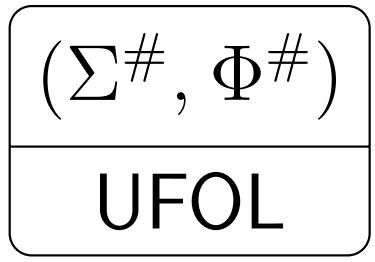
$\Sigma : (\{\mathsf{elem}, \mathsf{list}\}, \{\mathsf{nil}, \mathsf{cons}\}, \emptyset)$

Encodings

Example





Σ : $(\{\mathsf{elem}, \mathsf{list}\}, \{\mathsf{nil}, \mathsf{cons}\}, \emptyset)$
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Encodings

Example unary guard predicates for each erased sort

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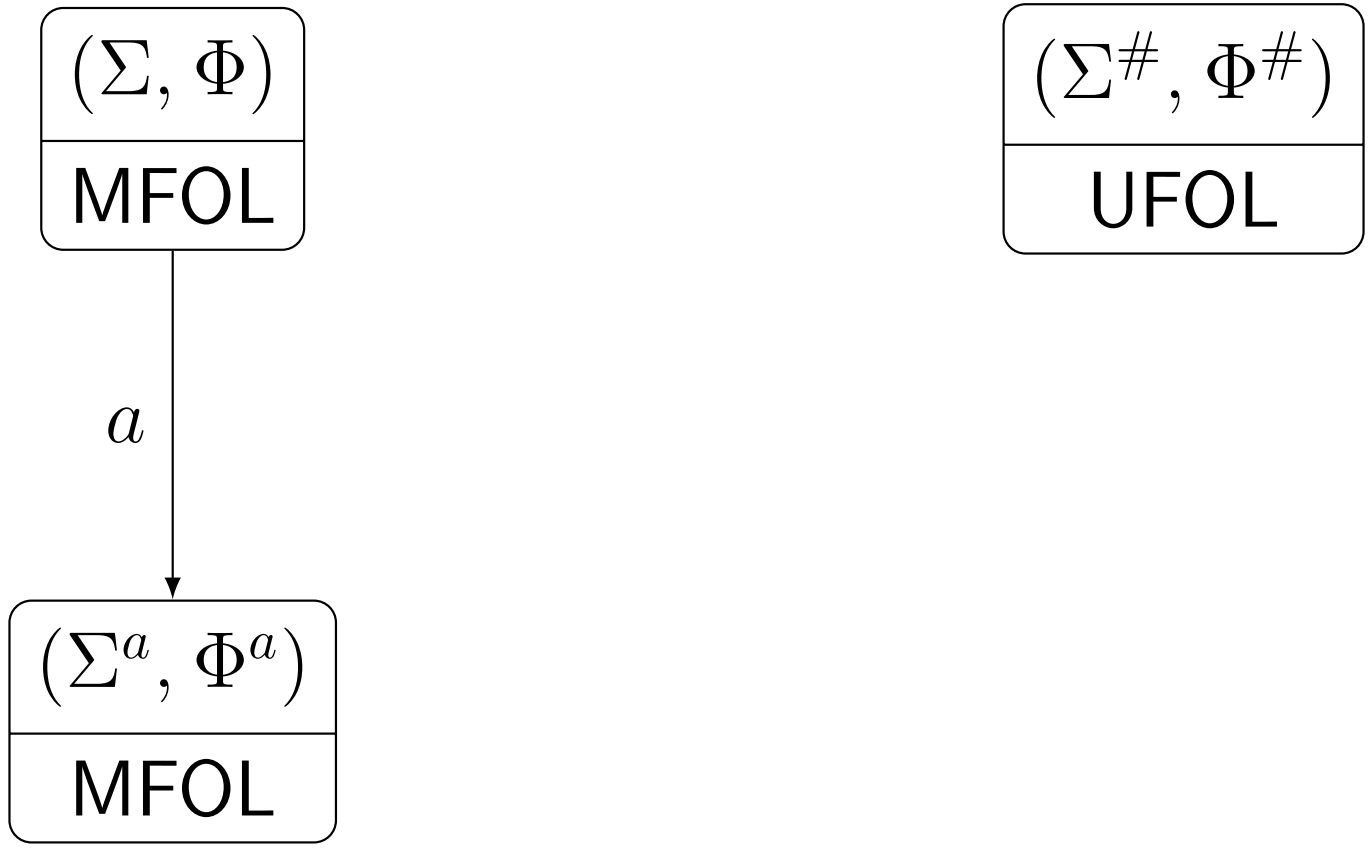
$g_{\mathsf{elem}}(x) \wedge g_{\mathsf{list}}(xs) \implies nil \neq cons(x, xs)$

Encodings

Methodology:

Encodings

Methodology: add decorations first,



Encodings

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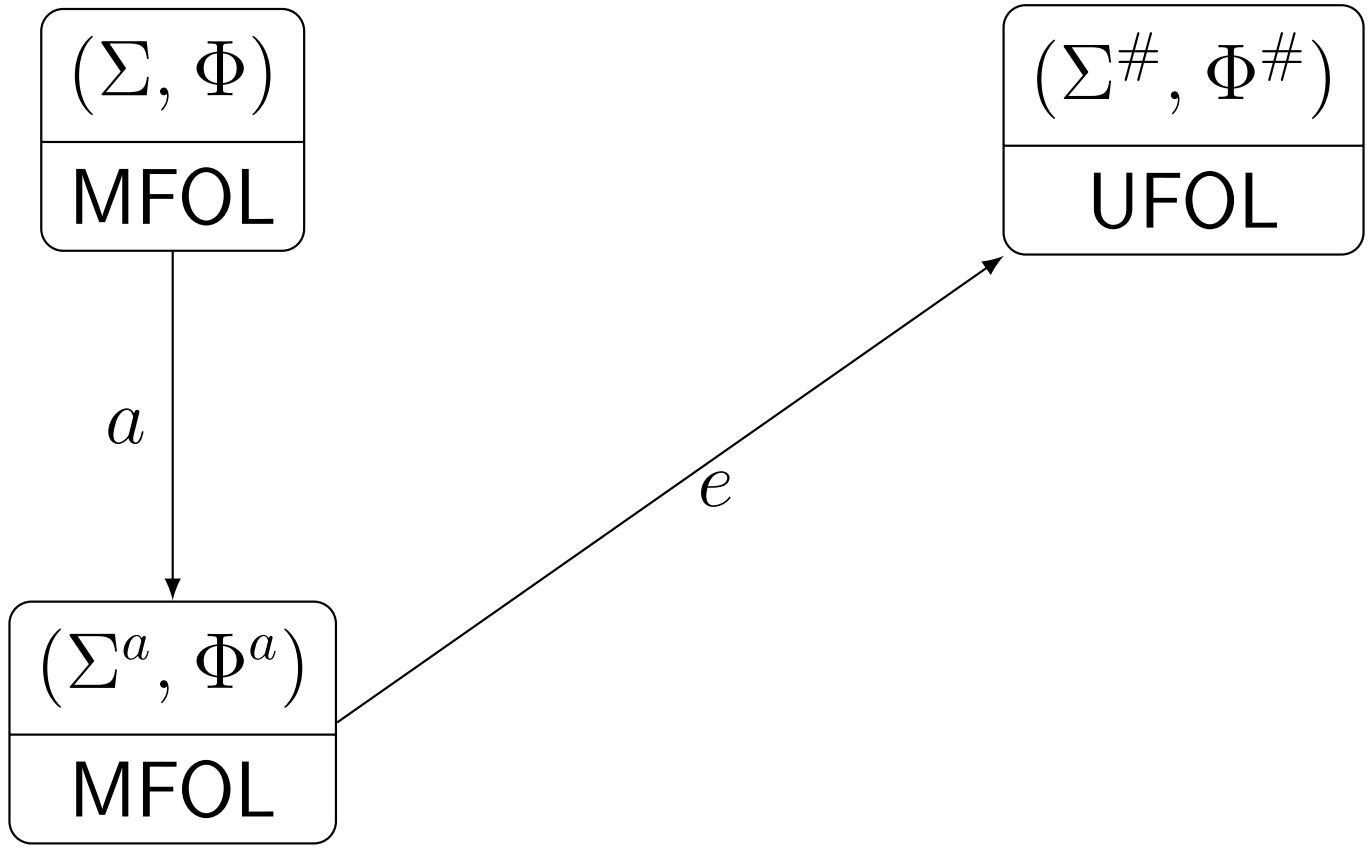
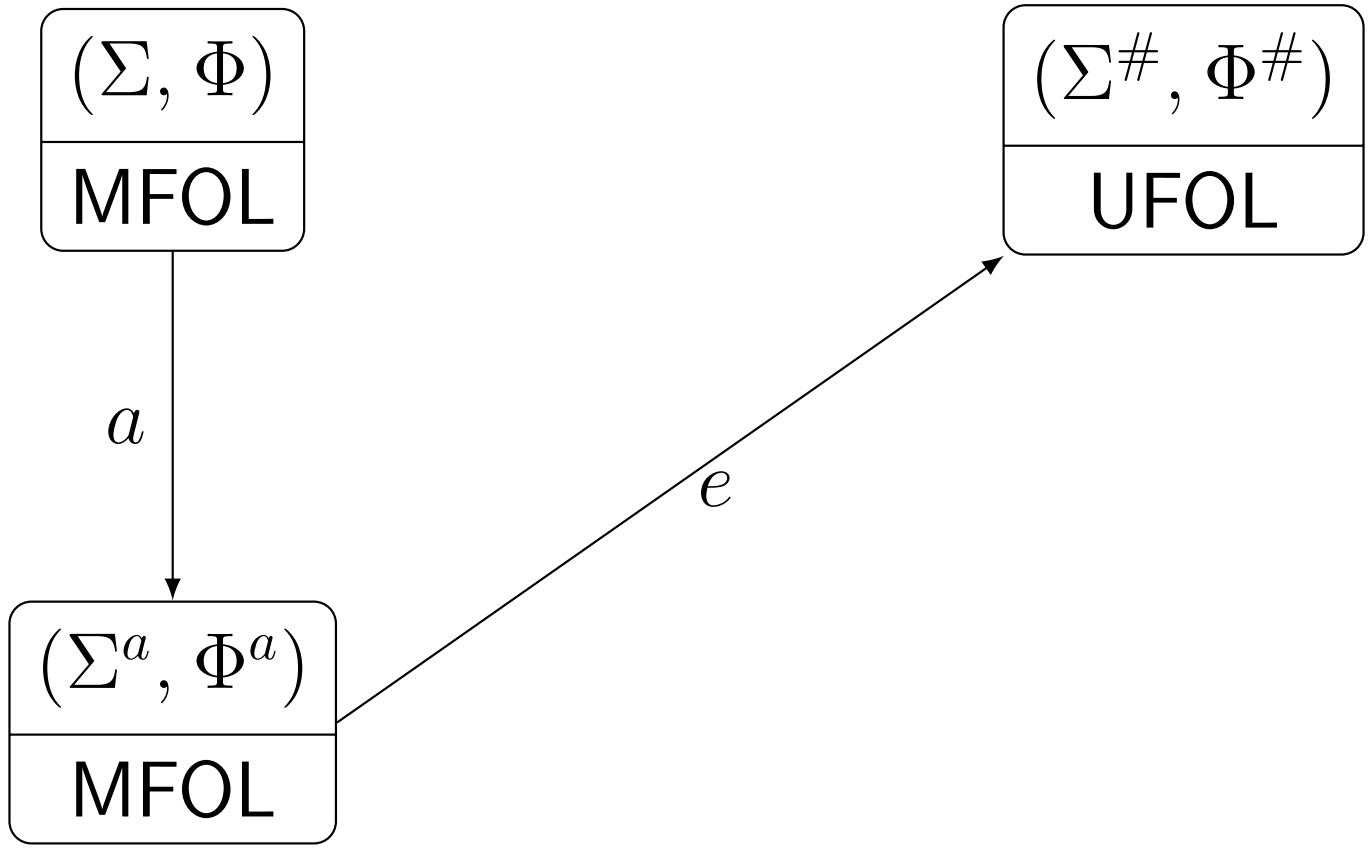


Encodings

Methodology: add decorations first, erase second

$$\# = e \circ a$$



Encodings

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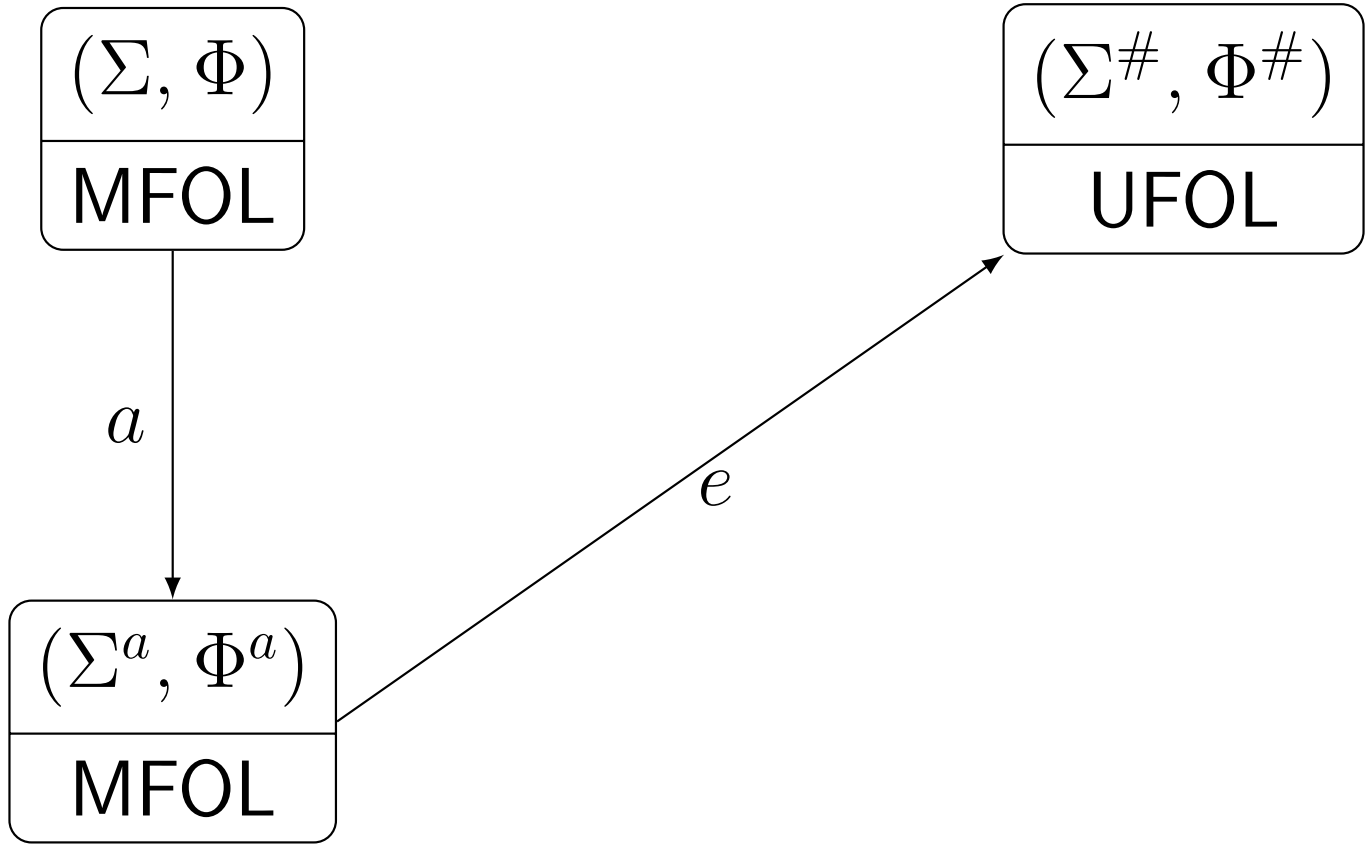
a needs to add enough decorations to make e sound



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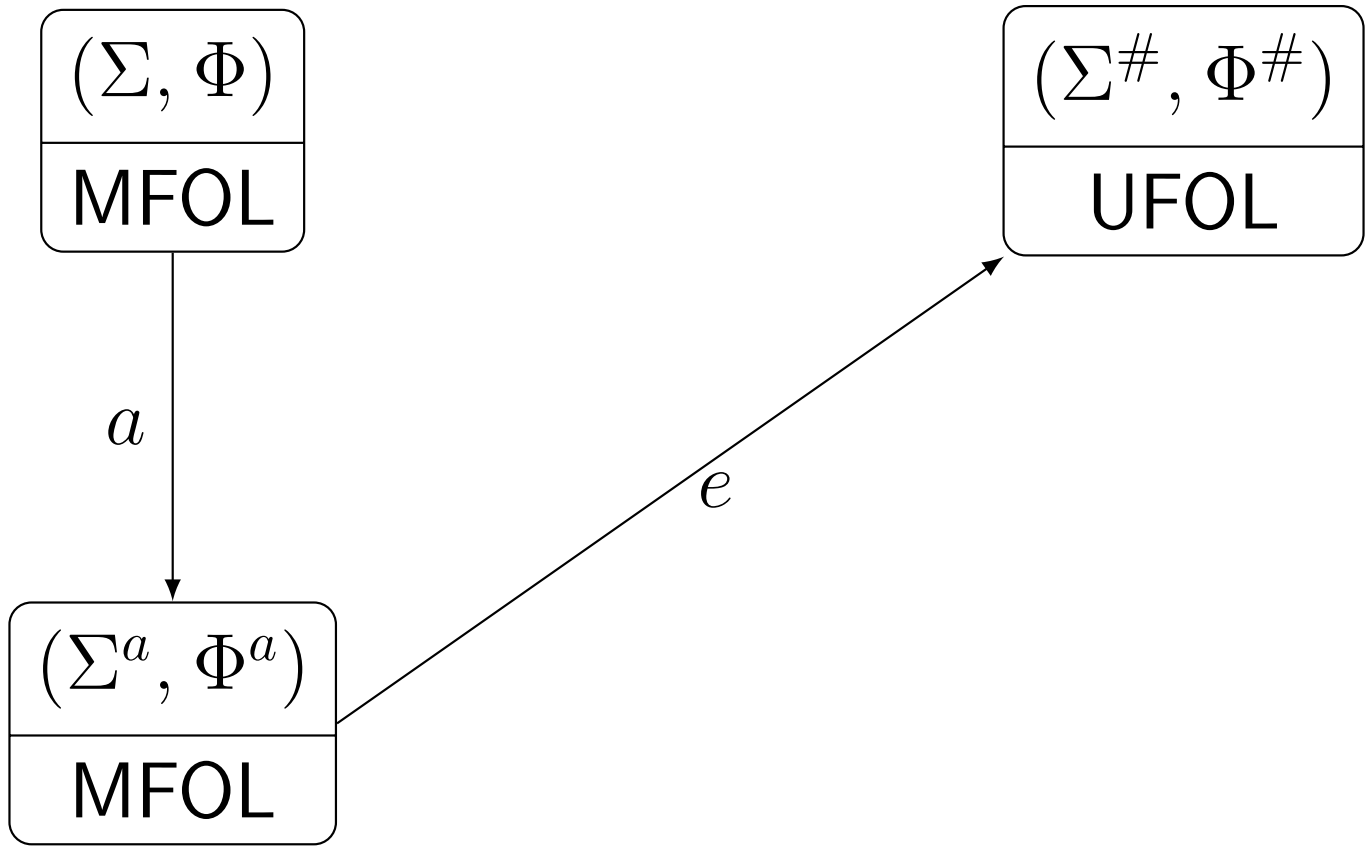
Sufficient: make Φ^a monotonic



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- Syntactic criteria

Our Formalization

- MFOL and UFOL
  - basics: formulas, models, satisfaction, substitution
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
- Type encodings from Blanchette et al.
- Monotonicity criteria from Claessen et al.
- Soundness and completeness

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MFOL_Prob $<_a$ MFOL_Mono_Prob $<_e$ UFOL_Prob

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  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
- Type encodings from Blanchette et al.
- Monotonicity criteria from Claessen et al.
- Soundness and completeness

Our Formalization

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Our Formalization

Can focus on at most countable models

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Our Formalization

Can focus on at most countable models
Use a fixed countable HOL type

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Our Formalization

Can focus on at most countable models
Use a fixed countable HOL type
Monotonicity analysis becomes manageable

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Our Formalization

- MFOL and UFOL
  - bureaucracy
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
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Our Formalization

New Isabelle theory of syntax with bindings

- MFOL and UFOL
  - eaucracy
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
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Our Formalization

New Isabelle theory of syntax with bindings
Recursors that interact well with substitution

- MFOL and UFOL
  - acy
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
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Our Formalization

New Isabelle theory of syntax with bindings
Recursors that interact well with substitution
Optimized for semantic interpretations of syntax

- MFOL and UFOL
  -
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
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Our Formalization

Formalized tableau-based proof a la Beth/Hintikka

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Our Formalization

Formalized tableau-based proof a la Beth/Hintikka
Infinite proof trees captured as a codatatype

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  - basics: formulas, models, satisfaction, substitution
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
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- Soundness and completeness

Our Formalization

Formalized tableau-based proof a la Beth/Hintikka
Infinite proof trees captured as a codatatype
Used Isabelle's codatatype package

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  - basics: formulas, models, satisfaction, substitution
  - classic results: completeness, compactness, Löwenheim-Skolem, clausification
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Conclusions

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- Have put Isabelle to work in order to verify its own communication with ATPs

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- Have put Isabelle to work in order to verify its own communication with ATPs
- Based on this, Isabelle might learn (how) to trust the ATPs

Mechanizing the Metatheory of Sledgehammer

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