MANCHESTER 1824

The University of Manchester

From Resolution and DPLL to Solving Arithmetic Constraints

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joint work with

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The University of Manchester

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Main Problem

Checking satisfiability of systems of linear inequalities over the rationals or reals.

Example 1

$$\begin{array}{rcl} 2x_1 - 4x_2 - 2x_3 - 2 & \geq & 0 \\ -x_1 + 2x_2 + 3x + 1 & \geq & 0 \\ 4x_2 + 2x_3 + 1 & = & 0 \end{array}$$

Satisfiable: $x_1 = 1/2, x_2 = -1/4, x_3 = 0$

Example 2

$$\begin{array}{rcrcrcc} -x_1 & + & x_2 & + & 1 & \geq & 0 \\ & - & x_2 & - & x_3 & \geq & 0 \\ x_1 & + & x_3 & - & 2 & \geq & 0 \end{array}$$

Unsatisfiable.

Solving Systems of Linear Inequalities

"Classical" methods:

- Fourier-Motzkin variable elimination method (Fourier, 1826);
- simplex (Dantzig, 1961);
- ellipsoid and interior point (Khachiyan 1979; Karmakar, 1984);

Methods inspired by propositional reasoning

- GDPLL [McMillan, Kuehlmann, Sagiv 2009]
- conflict resolution [Korovin, Tsiskaridze, Voronkov, 2009]
- UC-search [Cotton 2010]
- bound propagation [Korovin, Voronkov, 2011]
- cutting to the chase [Jovanović, de Moura, 2011]
- ...

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Propositional Logic vs Linear Arithmetic

propositional	linear arithmetic
clauses	linear inequalities
$\neg x_1 \vee x_2 \vee \cdots \vee x_n$	$-5x_1 + 3x_2 + \cdots + 0.5x_n + 17 \geq 0$
clause resolution	inequality resolution
$\dfrac{\neg x \vee C \quad x \vee D}{C \vee D}$	$\dfrac{-ax + p \geq 0 \quad bx + q \geq 0}{bp + aq \geq 0}$

Fourier-Motzkin elimination

Fourier-Motzkin Algorithm:

- Order variables: $x_n \succ \ldots \succ x_1$;
- Resolve:

$$\frac{-ax + p \geq 0 \quad bx + q \geq 0}{bp + aq \geq 0}$$

  x is the maximal variable in both inequalities.
- Eliminate variables by exhaustive resolution.

Main Issues with Fourier-Motzkin elimination

Inefficiencies of Fourier-Motzkin:

- Very quickly even a small system becomes unmanageably large;
- Generates many redundant inequalities;
- Even if a trivial solution exists F-M. performes all possible inferences.

Fourier-Motzkin

Example $x_5 \succ x_4 \succ x_3 \succ x_2 \succ x_1$

$$
\begin{array}{rcrcrcrcrcrcl}
2x_5 & - & 3x_4 & + & x_3 & - & 3x_2 & - & 2x_1 & + & 3 & \geq & 0 \\
2x_5 & + & x_4 & - & 2x_3 & & & - & 2x_1 & + & 2 & \geq & 0 \\
-x_5 & & & & & + & 3x_2 & + & x_1 & + & 2 & \geq & 0 \\
-3x_5 & & & + & 2x_3 & & & - & 3x_1 & - & 2 & \geq & 0 \\
x_5 & - & 2x_4 & & & - & 2x_2 & + & 3x_1 & - & 2 & \geq & 0 \\
-2x_5 & + & 2x_4 & - & 3x_3 & - & x_2 & + & 2x_1 & + & 3 & > & 0 \\
3x_5 & - & 2x_4 & + & 2x_3 & + & 3x_2 & + & 2x_1 & + & 1 & > & 0 \\
x_5 & & & & & & & + & 2x_1 & + & 2 & > & 0 \\
 & & 2x_4 & - & x_3 & - & 3x_2 & - & x_1 & + & 3 & = & 0
\end{array}
$$

Fourier-Motzkin: Generates over 280 million linear inequalities.

Proof search vs solution search

Observation: problems coming from applications usually:

- contain many constraints
- but only few are relevant in search for solution/contradiction

Main challenge: How to focus proof search to relevant constraints ?

Main idea: combine solution search and proof search

- originated in propositional reasoning: DPLL + lemma learning
- can we adapt this paradigm to other domains?

Note: orthogonal to DPLL(T)

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Conflict resolution

Main idea:

- start with a candidate solution – arbitrary assignment
- refine the candidate solution into a real solution
  - by deriving new constraints which are gradually correcting wrong variable assignments
- eventually either a solution will be found or contradiction derived.

Main properties:

- proof search is restricted to constraints which are in conflict with the candidate solution.
- solution search is restricted by the derived constraints

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Conflict Resolution

Fix: an ordering $x_n \succ \ldots \succ x_1$.

Goal: Find assignment of variables satisfying S.

Start with an arbitrary assignment: $\sigma = \{x_n \mapsto a_n, \ldots, x_1 \mapsto a_1\}$.

Apply:

- Assignment Refinement (AR)
- Conflict Resolution (CR)

until σ is refined to a solution or contradiction is derived.

Conflict Resolution

Example:

$$
\begin{array}{rcrcrcrcrcl}
x_4 & - & 2x_3 & & & + & x_1 & + & 5 & \geq & 0 \quad (1) \\
x_4 & + & 2x_3 & + & x_2 & & & + & 3 & \geq & 0 \quad (2) \\
-x_4 & - & x_3 & - & 3x_2 & - & 3x_1 & + & 1 & \geq & 0 \quad (3) \\
-x_4 & + & 2x_3 & + & 2x_2 & + & x_1 & + & 6 & \geq & 0 \quad (4) \\
 & & x_3 & & & + & 3x_1 & - & 1 & \geq & 0 \quad (5) \\
 & & -x_3 & + & x_2 & - & 2x_1 & + & 5 & \geq & 0 \quad (6)
\end{array}
$$

Conflict Resolution

$x_4 \succ x_3 \succ x_2 \succ x_1; \quad \sigma : \{x_4 \mapsto 0; x_3 \mapsto 0; x_2 \mapsto 0; x_1 \mapsto 0\}$

Level 4

$$(1) \quad 2x_3 - x_1 - 5 \leq x_4 \qquad x_4 \leq -x_3 - 3x_2 - 3x_1 + 1 \quad (3)$$

$$(2) \quad -2x_3 - x_2 - 3 \leq x_4 \qquad x_4 \leq 2x_3 + 2x_2 + x_1 + 6 \quad (4)$$

Level 3

$$(5) \quad -3x_1 + 1 \leq x_3 \qquad x_3 \leq x_2 - 2x_1 + 5 \quad (6)$$

Level 2 is empty

Level 1 is empty

Conflict Resolution

$x_4 \succ x_3 \succ x_2 \succ x_1; \quad \sigma : \{x_4 \mapsto 0; x_3 \mapsto 0; x_2 \mapsto 0; x_1 \mapsto 0\}$

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$$x_3 \in [1; 5]$$

Level 2 is empty

Level 1 is empty

Conflict Resolution

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Level 3

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$$x_3 \in [1;5] \; x_3 = 4 \qquad (AR)$$

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Level 1 is empty

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$x_4 \succ x_3 \succ x_2 \succ x_1; \quad \sigma : \{x_4 \mapsto 0; x_3 \mapsto 4; x_2 \mapsto 0; x_1 \mapsto 0\}$

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$$x_4 \in [3; -3]$$

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$$x_3 \in [1; 5] \; x_3 = 4 \qquad (AR)$$

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$$x_4 \in [3; -3] \quad \textit{Conflict!}$$

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Level 4

(1) $2x_3 - x_1 - 5 \leq x_4$ $\qquad$ $x_4 \leq -x_3 - 3x_2 - 3x_1 + 1$ (3)

(2) $-2x_3 - x_2 - 3 \leq x_4$ $\qquad$ $x_4 \leq 2x_3 + 2x_2 + x_1 + 6$ (4)

$x_4 \in [3; -3]$ *Conflict*!

Level 3

(5) $-3x_1 + 1 \leq x_3$ $\qquad$ $x_3 \leq x_2 - 2x_1 + 5$ (6)

$x_3 \leq -x_2 - \frac{2}{3}x_1 + 2$ (CR)

$x_3 \in [1; 5]$ $x_3 = 4$ (AR)

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Satisfiable! $\sigma : \{x_4 \mapsto 0; x_3 \mapsto 1, x_2 \mapsto 0; x_1 \mapsto 0\}$

Conflict Resolution

Conflict Resolution Algorithm (CRA) is correct and terminating.

Theorem. Let S be a set of linear constraints then:

- S is unsatisfiable iff CRA derives $0 \geq 1$;
- S is satisfiable iff CRA terminates with a satisfying assignment.

[Conflict Resolution; Korovin, Tsiskaridze, Voronkov; CP'09]

Properties of CRA

Redundancy. A CR-inference with premises at a level k is redundant if its conclusion follows from $S_{<k}$.

Properties of CRA.

- Every CR-inference performed by CRA is non-redundant;
- In particular, the same constraint is never be added twice;
- CRA is exponentially more efficient than F-M (independently of initial assignments) on a class of problems.

Properties for SMT integration.

- CRA can be easily made incremental;
- CRA can easily generate explanations for unsatisfiability.

[Implementing Conflict Resolution; Korovin, Tsiskaridze, Voronkov; PSI'11]

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Conflict Resolution: Generates 21 linear inequalities.

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From conflict resolution to bound propagation

Conflict resolution: Combines solution and proof search for linear arithmetic.

What is missing?

- unit propagation
- backjumping
- dynamic variable ordering

Next: Bound Propagation.

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Bounds

Unit constraints: bounds on variables.

Example: $x \geq 0$, $-x \geq -2/3$.

Bound propagation: derive new bounds by resolving inequalities with current bounds.

$$\frac{\dfrac{x_4 \geq 1 \quad x_3 - x_4 \geq -1}{x_3 \geq 0} \qquad -x_2 \geq 0 \quad x_4 \geq 1 \quad x_1 + x_2 - x_3 - x_4 \geq 0}{x_1 \geq 1}$$

$x_1 \geq 1$ is derived by bound propagation from

- bounds $\{-x_2 \geq 0, x_4 \geq 1\}$ and
- inequalities $\{x_3 - x_4 \geq -1, x_1 + x_2 - x_3 - x_4 \geq 0\}$

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- bounds $\{-x_2 \geq 0, x_4 \geq 1\}$ and
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Example: Bound Propagation

$$\begin{array}{rcr} -y & \geq & -1 \\ x & \geq & 1 \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \end{array}$$

inequalities

Example: Bound Propagation

$$\begin{array}{rcr} -y & \geq & -1 \\ x & \geq & 1 \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \end{array}$$

inequalities

Example: Bound Propagation

$$\begin{array}{rcr} -z & \geq & -1/2 \\ -y & \geq & -1 \\ x & \geq & 1 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \\ \hline \end{array}$$

inequalities

Example: Bound Propagation

$$\begin{array}{rcr} -z & \geq & -1/2 \\ -y & \geq & -1 \\ x & \geq & 1 \end{array}$$

bounds

$$\begin{array}{rcr} y+2z-x & \geq & 1 \\ 4z+y+x & \geq & 5 \\ -x+2y-2z & \geq & 0 \end{array}$$

inequalities

Example: Bound Propagation

$$\begin{array}{rcr} x & \geq & 2 \\ -z & \geq & -1/2 \\ -y & \geq & -1 \\ x & \geq & 1 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \\ \hline \end{array}$$

inequalities

Example: Bound Propagation

$$\begin{array}{rcr} x & \geq & 2 \\ -z & \geq & -1/2 \\ -y & \geq & -1 \\ x & \geq & 1 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \\ \hline \end{array}$$

inequalities

Example: Bound Propagation

$$\begin{array}{rcr} 0 & \geq & 1 \\ x & \geq & 2 \\ -z & \geq & -1/2 \\ -y & \geq & -1 \\ x & \geq & 1 \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \end{array}$$

inequalities

Unsatisfiable!

Example: Bound Propagation

$$\begin{array}{rcr} 0 & \geq & 1 \\ x & \geq & 2 \\ -z & \geq & -1/2 \\ -y & \geq & -1 \\ x & \geq & 1 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcr} y + 2z - x & \geq & 1 \\ 4z + y + x & \geq & 5 \\ -x + 2y - 2z & \geq & 0 \\ \hline \end{array}$$

inequalities

Unsatisfiable!

Note: different bounds on the same variable
Contradiction obtained with the improved bound.

Bound propagation

$$\frac{x_1 \geq 0}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation

$$\frac{x_1 \geq 0}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation

$$\frac{\begin{array}{c} x_2 \geq 0 \\ x_1 \geq 0 \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation

$$\frac{x_2 \geq 0 \quad x_1 \geq 0}{\text{bounds}}$$

$$\frac{x_2 - x_1 \geq 0 \quad x_1 - x_2 \geq 1}{\text{inequalities}}$$

Bound propagation

$$\frac{\begin{array}{c} x_1 \geq 1 \\ x_2 \geq 0 \\ x_1 \geq 0 \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation

$$\frac{\begin{array}{c} x_2 \geq 1 \\ x_1 \geq 1 \\ x_2 \geq 0 \\ x_1 \geq 0 \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation

$$\frac{\begin{array}{c} x_1 \geq 2 \\ x_2 \geq 1 \\ x_1 \geq 1 \\ x_2 \geq 0 \\ x_1 \geq 0 \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation

$$\frac{\begin{array}{c} x_2 \geq 2 \\ x_1 \geq 2 \\ x_2 \geq 1 \\ x_1 \geq 1 \\ x_2 \geq 0 \\ x_1 \geq 0 \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Bound propagation is non-terminating

$$\frac{\begin{array}{c} \dots \\ x_2 \geq 2 \\ x_1 \geq 2 \\ x_2 \geq 1 \\ x_1 \geq 1 \\ x_2 \geq 0 \\ x_1 \geq 0 \end{array}}{\text{bounds}} \qquad \frac{\begin{array}{c} x_2 - x_1 \geq 0 \\ x_1 - x_2 \geq 1 \end{array}}{\text{inequalities}}$$

Another Cycle

$$x \geq 0$$

bounds

$$\begin{array}{rcl} -x + y & \geq & 0 \\ -y + 2x & \geq & 1 \end{array}$$

inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Another Cycle

y	$\geq$	0
x	$\geq$	0

bounds

$-x + y$	$\geq$	0
$-y + 2x$	$\geq$	1

inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Another Cycle

$$\begin{array}{ccr} x & \geq & 1/2 \\ y & \geq & 0 \\ x & \geq & 0 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcl} -x+y & \geq & 0 \\ -y+2x & \geq & 1 \\ \hline \end{array}$$

inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Another Cycle

$$\begin{array}{lcr} y & \geq & 1/2 \\ x & \geq & 1/2 \\ y & \geq & 0 \\ x & \geq & 0 \end{array}$$

bounds

$$\begin{array}{rcl} -x+y & \geq & 0 \\ -y+2x & \geq & 1 \end{array}$$

inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Another Cycle

$$\begin{array}{rcr} & \dots & \\ x & \geq & 1/2+1/4+\dots+1/2n \\ & \dots & \\ x & \geq & 1/2+1/4 \\ y & \geq & 1/2 \\ x & \geq & 1/2 \\ y & \geq & 0 \\ x & \geq & 0 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcl} -x+y & \geq & 0 \\ -y+2x & \geq & 1 \\ \hline \end{array}$$

inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Another Cycle

$$\begin{array}{ccr}
 & \dots & \\
x & \geq & 1/2 + 1/4 + \dots + 1/2n \\
 & \dots & \\
x & \geq & 1/2 + 1/4 \\
y & \geq & 1/2 \\
x & \geq & 1/2 \\
y & \geq & 0 \\
x & \geq & 0 \\
\hline
\end{array}
\qquad
\begin{array}{rcc}
-x + y & \geq & 0 \\
-y + 2x & \geq & 1 \\
\hline
\end{array}$$

bounds — inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Another Cycle

$$\begin{array}{ccr} & \dots & \\ x & \geq & 1/2+1/4+\ldots+1/2n \\ & \dots & \\ x & \geq & 1/2+1/4 \\ y & \geq & 1/2 \\ x & \geq & 1/2 \\ y & \geq & 0 \\ x & \geq & 0 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcc} -x+y & \geq & 0 \\ -y+2x & \geq & 1 \\ \hline \end{array}$$

inequalities

Bounds on x are approaching their limit 1 but never reach it.

Cycles: a bound on a variable is used to improve a bound on the same variable.

Eliminate cycles by using collapsing inequalities.

Collapsing Inequalities

For any cycle we can derive a collapsing inequality I such that

- I is implied by our set of inequalities
- using I we can derive the bound that can not be improved further by the cycle.

The proof is based on Farkas theorem.

Why decisions are necessary

Bound propagation is insufficient even if all variables are bounded.

Example:

$$\begin{array}{ccccc} 1 & \geq & y & \geq & 0 \\ 1 & \geq & x & \geq & 0 \end{array}$$

bounds

$$\begin{array}{rcr} x - y & \geq & 0 \\ y - x & \geq & 0 \\ x + y & \geq & 1 \\ -x - y & \geq & -1 \end{array}$$

inequalities

Solution: $x = y = 1/2$ but no new bound is derivable by bound propagation.

Why decisions are necessary

Bound propagation is insufficient even if all variables are bounded.

Example:

$$\begin{array}{ccccc} & & x & := & 1/3 \\ 1 & \geq & y & \geq & 0 \\ 1 & \geq & x & \geq & 0 \\ \hline \end{array}$$

bounds

$$\begin{array}{rcr} x - y & \geq & 0 \\ y - x & \geq & 0 \\ x + y & \geq & 1 \\ -x - y & \geq & -1 \\ \hline \end{array}$$

inequalities

Solution: $x = y = 1/2$ but no new bound is derivable by bound propagation.

Why decisions are necessary

Bound propagation is insufficient even if all variables are bounded.

Example:

$$
\begin{array}{ccccc}
 & & 0 \geq 1 & & \\
 & & \cdots & & \\
 & & x & := 1/3 & \\
1 & \geq & y & \geq & 0 \\
1 & \geq & x & \geq & 0 \\
\hline
 & & \text{bounds} & &
\end{array}
\qquad
\begin{array}{rcr}
x - y & \geq & 0 \\
y - x & \geq & 0 \\
x + y & \geq & 1 \\
-x - y & \geq & -1 \\
\hline
 & \text{inequalities} &
\end{array}
$$

Solution: $x = y = 1/2$ but no new bound is derivable by bound propagation.

Why decisions are necessary

Bound propagation is insufficient even if all variables are bounded.

Example:

$$
\begin{array}{ccccc}
 & & 0 \geq 1 & & \\
 & & \cdots & & \\
 & & x & := 1/3 & \\
1 & \geq & y & \geq & 0 \\
1 & \geq & x & \geq & 0 \\
\hline
\multicolumn{5}{c}{\text{bounds}}
\end{array}
\qquad
\begin{array}{rcr}
x - y & \geq & 0 \\
y - x & \geq & 0 \\
x + y & \geq & 1 \\
-x - y & \geq & -1 \\
\hline
\multicolumn{3}{c}{\text{inequalities}}
\end{array}
$$

Solution: $x = y = 1/2$ but no new bound is derivable by bound propagation.

Collapsing inequality: $x \geq 1/2$.

Algorithm (informal and simplified)

1. Apply (limited) bound propagation;
2. If there are no bounds or limit is reached assign a value to an unassigned variable within the current bounds for this variable.
3. If a solution is found, return the solution.
4. If an inconsistent context/cycle is obtained, generate a collapsing inequality.
   - Use this inequality to derive a bound improving a previously asserted bound and backjump.
5. If derive inconsistent bounds at level 0, return unsatisfiable.

Example

$$\frac{\text{level 0}}{\text{bounds}}$$

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \\ \hline \end{array}$$

inequalities

Example

$$\frac{\begin{array}{c} x_0 := 0 \\ \text{level } 1 \\ \text{level } 0 \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}}{\text{inequalities}}$$

Example

$$\frac{\begin{array}{ccc} x_0 & := & 0 \\ & \text{level } 1 & \\ & \text{level } 0 & \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}}{\text{inequalities}}$$

Example

$$\begin{array}{ccc} x_1 & \geq & 0 \\ x_0 & := & 0 \\ & \text{level } 1 & \\ & \text{level } 0 & \\ \hline & \text{bounds} & \end{array}$$

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \\ \hline & \text{inequalities} & \end{array}$$

Example

$$\frac{\begin{array}{ccc} x_1 & \geq & 0 \\ x_0 & := & 0 \\ & \text{level } 1 & \\ & \text{level } 0 & \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}}{\text{inequalities}}$$

Example

$$\begin{array}{ccc} x_0 & \geq & 1 \\ x_1 & \geq & 0 \\ x_0 & := & 0 \end{array}$$

level 1

level 0

bounds

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}$$

inequalities

Example

contradiction
$x_0 \geq 1$
$x_1 \geq 0$
$x_0 := 0$
level 1
level 0
bounds

$$\begin{aligned} x_0 - 2x_1 &\geq 1 \\ x_0 + 2x_1 &\geq 1 \\ -x_0 + x_1 &\geq 0 \end{aligned}$$

inequalities

Example

$$\begin{array}{ccc} \multicolumn{3}{c}{\text{contradiction}} \\ x_0 & \geq & 1 \\ x_1 & \geq & 0 \\ x_0 & := & 0 \\ \multicolumn{3}{c}{\text{level 1}} \\ \multicolumn{3}{c}{\text{level 0}} \\ \hline \multicolumn{3}{c}{\text{bounds}} \end{array}$$

$$\begin{array}{rcc} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \\ \hline \multicolumn{3}{c}{\text{inequalities}} \end{array}$$

Collapsing inequality:

$$\frac{x_0 - 2x_1 \geq 1 \quad -x_0 + x_1 \geq 0}{-x_0 \geq 1}$$

Example

bounds	
contradiction	
$x_0 \geq 1$	
$x_1 \geq 0$	
$x_0 := 0$	
level 1	
level 0	

inequalities
$x_0 - 2x_1 \geq 1$
$x_0 + 2x_1 \geq 1$
$-x_0 + x_1 \geq 0$

Collapsing inequality:

$$\frac{x_0 - 2x_1 \geq 1 \quad -x_0 + x_1 \geq 0}{-x_0 \geq 1}$$

The obtained bound $-x_0 \geq 1$ contradicts to the asserted bound $x_0 = 0$, so we backjump and remove this assertion.

Example, continued

$$\frac{\begin{array}{rcl} -x_0 & \geq & 1 \\ \multicolumn{3}{c}{\text{level } 0} \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}}{\text{inequalities}}$$

Example, continued

$$\frac{\begin{array}{rcl} -x_0 & \geq & 1 \\ & \text{level } 0 & \end{array}}{\text{bounds}}$$

$$\frac{\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}}{\text{inequalities}}$$

Example, continued

$$\begin{array}{rcl} -x_1 & \geq & 1 \\ -x_0 & \geq & 1 \\ \multicolumn{3}{c}{\text{level } 0} \end{array}$$

bounds

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}$$

inequalities

Example, continued

$$\begin{array}{rcl} -x_1 & \geq & 1 \\ -x_0 & \geq & 1 \\ \multicolumn{3}{c}{\text{level } 0} \\ \hline \end{array}$$

bounds

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \\ \hline \end{array}$$

inequalities

Example, continued

$$\begin{array}{rcl} 0 & \geq & 4 \\ -x_1 & \geq & 1 \\ -x_0 & \geq & 1 \\ \multicolumn{3}{c}{\text{level } 0} \end{array}$$

bounds

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}$$

inequalities

Example, continued

contradiction

$$\begin{array}{rcl} 0 & \geq & 4 \\ -x_1 & \geq & 1 \\ -x_0 & \geq & 1 \end{array}$$

level 0

bounds

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}$$

inequalities

Example, continued

contradiction

$$\begin{array}{rcl} 0 & \geq & 4 \\ -x_1 & \geq & 1 \\ -x_0 & \geq & 1 \end{array}$$

level 0

bounds

$$\begin{array}{rcl} x_0 - 2x_1 & \geq & 1 \\ x_0 + 2x_1 & \geq & 1 \\ -x_0 + x_1 & \geq & 0 \end{array}$$

inequalities

Since the contradiction is obtained at level 0, the system is unsatisfiable.

Bound propagation

Theorem. Bound propagation algorithm is
sound, complete and terminating.

Bound propagation combines solution and proof search and features:

- unit propagation
- dynamic variable ordering
- lemma learning
- backjumping

[Solving Systems of Linear Inequalities by Bound Propagation;
Korovin, Voronkov, CADE'11]

Implementation

Key parameters of bound propagation:

- variable selection: tightest bound, VSIDS,...
- value selection: continued fraction approximations, ...
- conflict selection: maximal overlap, relaxation, ...

Performance:

- overall not far behind optimized simplex implementations
- can solve problems that optimized simplex-based solvers such as Z3 cannot.

[Bound Propagation for Arithmetic Reasoning in Vampire; Dragan, Korovin, Kovács, Voronkov; SYNASC'13]

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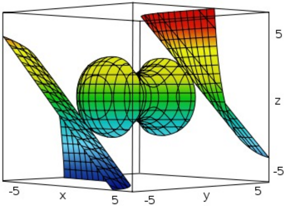
Recent advances and future directions

The paradigm of combining solution and proof search in the DPLL style reasoning can be fruitfully applied to many domains.

- rational linear arithmetic
- integer linear arithmetic
- polynomial constraints over reals: combination of cylindrical cell decomposition with DPLL. Outperforms all other systems for solving non-linear inequalities including Mathematica. [Jovanović, de Moura]

Future directions:

- complex domains: non-linear and integers
- integration of arithmetic into first-order reasoning



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