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Non-cyclic sorts for first-order satisfiability

(or how to win first-order satisfiability at CASC)

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First-order satisfiability

The problem: Given a set of first-order sentences S check whether S is satisfiable.

Complementary to proof finding: Given a set of first-order sentences check whether it is unsatisfiable.

Where satisfiability checking is used?

- in verification for finding errors in systems
- in combinatorial reasoning: scheduling, planning, etc. for finding solutions
- in checking consistency of ontologies, theories, axiomatisations
- disproving conjectures
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Methods for finite model finding

General first-order satisfiability is not recursively enumerable.

Restrict to finite model finding (FMF).

- Finite model finding is recursively enumerable.
- But usual first-order reasoning methods such resolution/superposition are incomplete for finite model finding

Methods for finite model finding are based on encodings into:

- Propositional logic (FINDER, MACE, Paradox)
  [Slaney; McCune; Claessen, Sörensson]
  Paradox has been winning satisfiability at CASC for the last 10 years.
- Geometric logic (Geo) [de Nivelle, Meng]
- Effectively propositional logic (EPR) (DarwinFM, iProver)
  [Baumgartner, de Nivelle, Fuchs, Tinelli]

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Effectively Propositional Logic (EPR)

EPR: No functions except constants: $P(x, y) \vee \neg Q(c, y)$

Transitivity: $\neg P(x, y) \vee \neg P(y, z) \vee P(x, z)$

Symmetry: $P(x, y) \vee \neg P(y, x)$

Verification:

$$\forall A(\mathtt{wren}_{h1} \wedge A = \mathtt{wraddrFunc} \rightarrow \forall B(\mathtt{range}_{[35,0]}(B) \rightarrow (\mathtt{imem}'(A, B) \leftrightarrow \mathtt{iwrite}(B)))).$$

Applications many problems can be encoded into the EPR:

- Hardware Verification (Intel)
- Planning/Scheduling
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Instantiation-based methods excel in the EPR fragment.

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Basic idea: Eliminate functions

- $C[t] \Rightarrow t \not\simeq x \vee C[x]$
- $Q(f(g(x)))$

$$\Rightarrow \neg P_f(y_1, y_2) \vee \neg P_g(x, y_1) \vee Q(y_2)$$

Step2. Replace functions by predicates:

- $f(x_1, \ldots, x_n) \simeq y$ can be represented by $P_f(x_1, \ldots, x_n, y)$ provided:
- P_f is right-unique:

$$\forall \bar{x}, y[(P_f(\bar{x}, y) \wedge P_f(\bar{x}, y')) \rightarrow y \simeq y']$$

function-free EPR (possible to drop)

- P_f right-total:

$$\forall \bar{x} \exists y P_f(\bar{x}, y)$$

for finite domains can be expressed using domain axiom:

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EPR-based finite model finding [Baumgartner, de Nivelle, Fuchs, Tinelli]

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Is flattening always good ?

Flattening

- essential for getting minimal wrt. size models
- but flattening can be bad for performance of reasoning systems

Example: trivial propositional problem

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\begin{array}{ccccc}
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Domain k: $P_{c_1}(1) \vee \ldots \vee P_{c_1}(k) \quad \ldots \quad P_{c_n}(1) \vee \ldots \vee P_{c_n}(k)$

Non-trivial after flattening. Next: how to avoid unnecessary flattening

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Extension EPR into many-sorted logic

Observe: if a problem is EPR we do not need to apply flattening.

Can we do more?

EPR is decidable because:

- the set of all ground terms (the Herbrand universe) is finite

In unsorted first-order logic Herbrand universe is finite if and only if all function symbols are constants.

Observation: [Abadi, Rabinovich, Sagiv] In the presence of sorts the Herbrand universe can be finite even in the presence of non-constant function symbols, under certain conditions.

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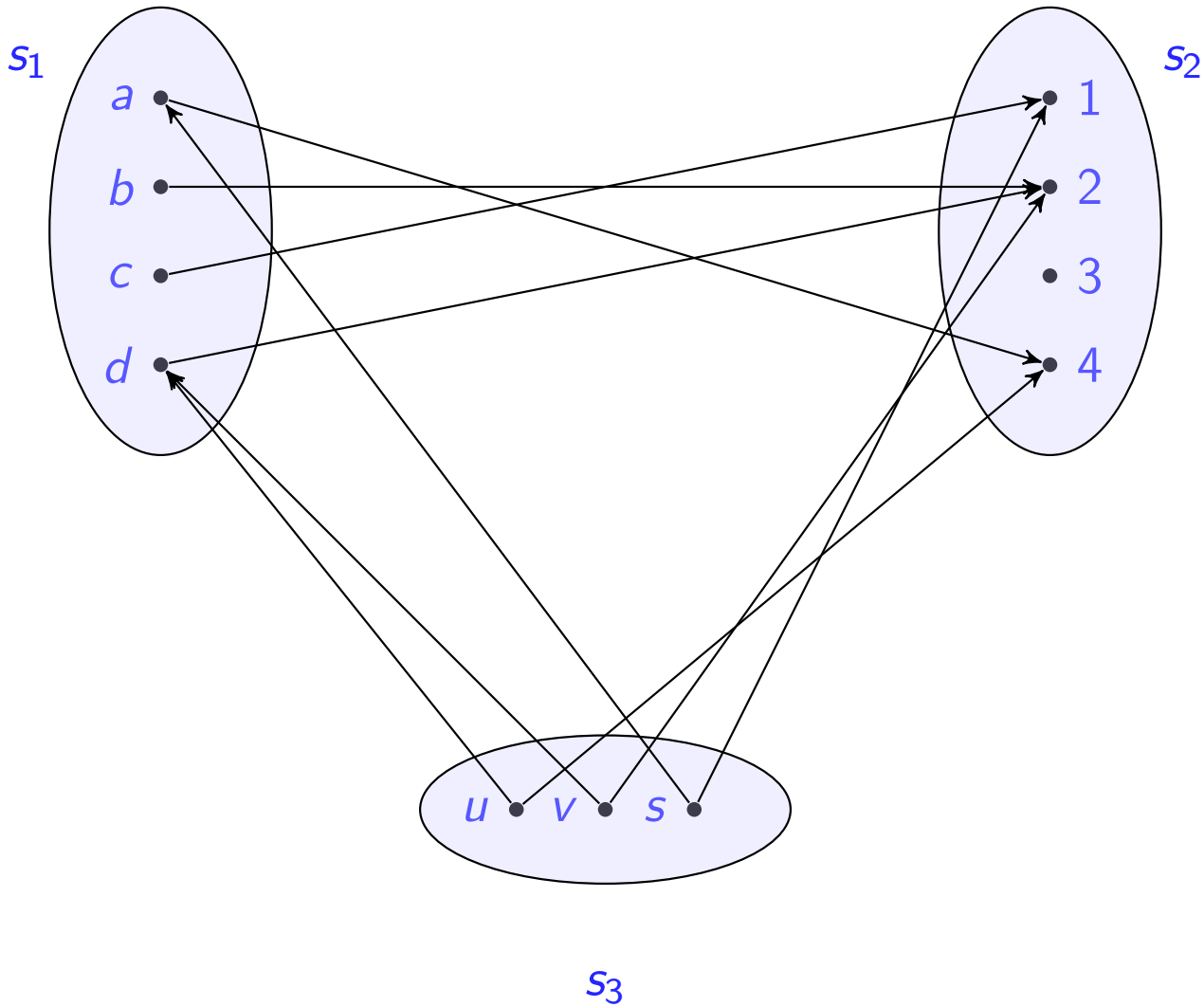

Restriction: no cyclic dependencies

Non-cyclic fragment

Consider: a signature $\Sigma = \langle \mathcal{S}, \mathcal{F}, \mathcal{P} \rangle$

- A sort dependency graph $SD(\Sigma) = \langle \mathcal{S}, \rightarrow \rangle$
  where $s_i \rightarrow s$ iff there is $f \in \mathcal{F}$ such that
  $$f : s_1 \times \ldots \times s_i \times \ldots \times s_n \mapsto s$$
- A signature is non-cyclic if there are no cycles in its sort dependency graph.
- The non-cyclic clausal fragment consists of sets of clauses over a non-cyclic signature.

Proposition. In many-sorted logic the Herbrand universe is finite if and only if the signature is non-cyclic.

Theorem. Non-cyclic fragment is decidable by instantiation based-methods.

Theorem. There is a linear-time algorithm for checking whether a clause set is in the non-cyclic fragment.

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Unfortunately:

- Many problems are almost in the non-cyclic fragment.

Main idea: decompose the sort dependency graph into

- cyclic and non-cyclic sorts.

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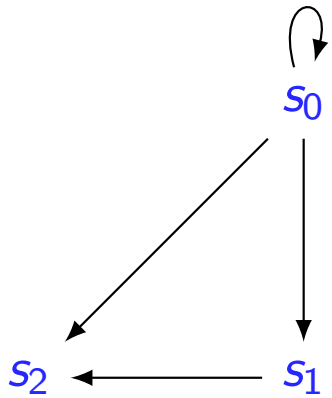
Sort decomposition

Non-cyclic decomposition: Decompose all sorts into cyclic and non-cyclic.

Sort dependency graph



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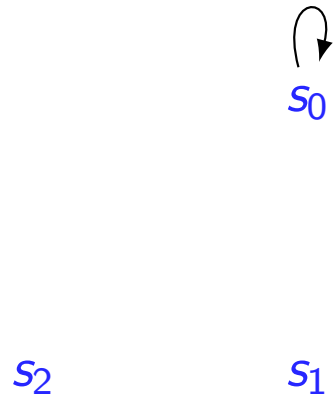

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The proof is based on Tarjan's algorithm for decomposing directed graphs into strongly connected components.

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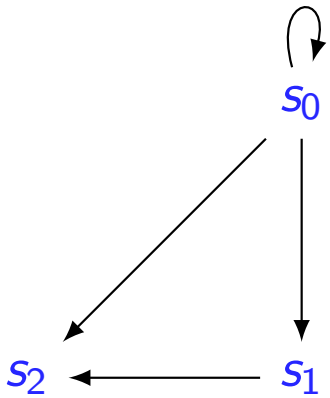

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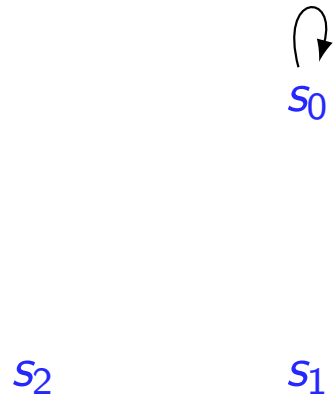

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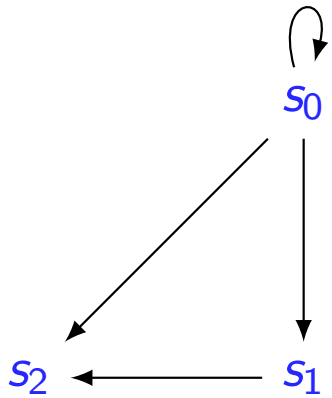

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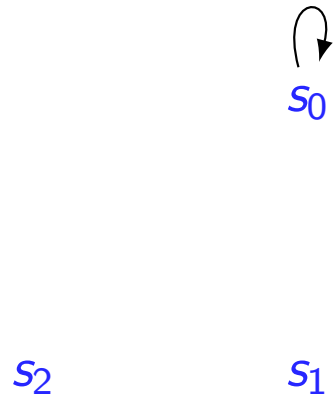

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Sort-restricted finite model finding

Consider:

- A set of clauses S over a signature Σ and
- the non-cyclic decomposition of Σ.

Sort-restricted finite model finding:

- restrict flattening to terms of cyclic sorts,
- apply instantiation-based methods to the obtained non-cyclic clauses

Theorem. Sort-restricted finite-model finding is complete for finite satisfiability.

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Note: Cyclic and non-cyclic sorts can be interleaved inside a term.

Consider: a signature with function symbols f, g, h, c with one cyclic sort: $\mathit{valSort}(g) = \mathit{argSort}(1, g) = \mathit{argSort}(h, 1) = s$

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Domain : $P_g(x, y, 1) \vee \ldots \vee P_g(x, y, n)$

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TPTP benchmarks: $15,550$ first-order problems.

Unfortunately: all these problems are unsorted.

Solution:

- infer sorts automatically (linear-time)

  [Claessen, Sörensson; Claessen, Lilliestrom, Smallbone]
- $4,090$ problems have more than one inferred sort.

Non-cyclic/EPR sorts in TPTP

- $1,383$ pure EPR problems
- $2,578$ problems have at least one non-cyclic sort
- $56,679$ collective number of sorts
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Summary: Non-cyclic sorts are in more than half of sortified problems.

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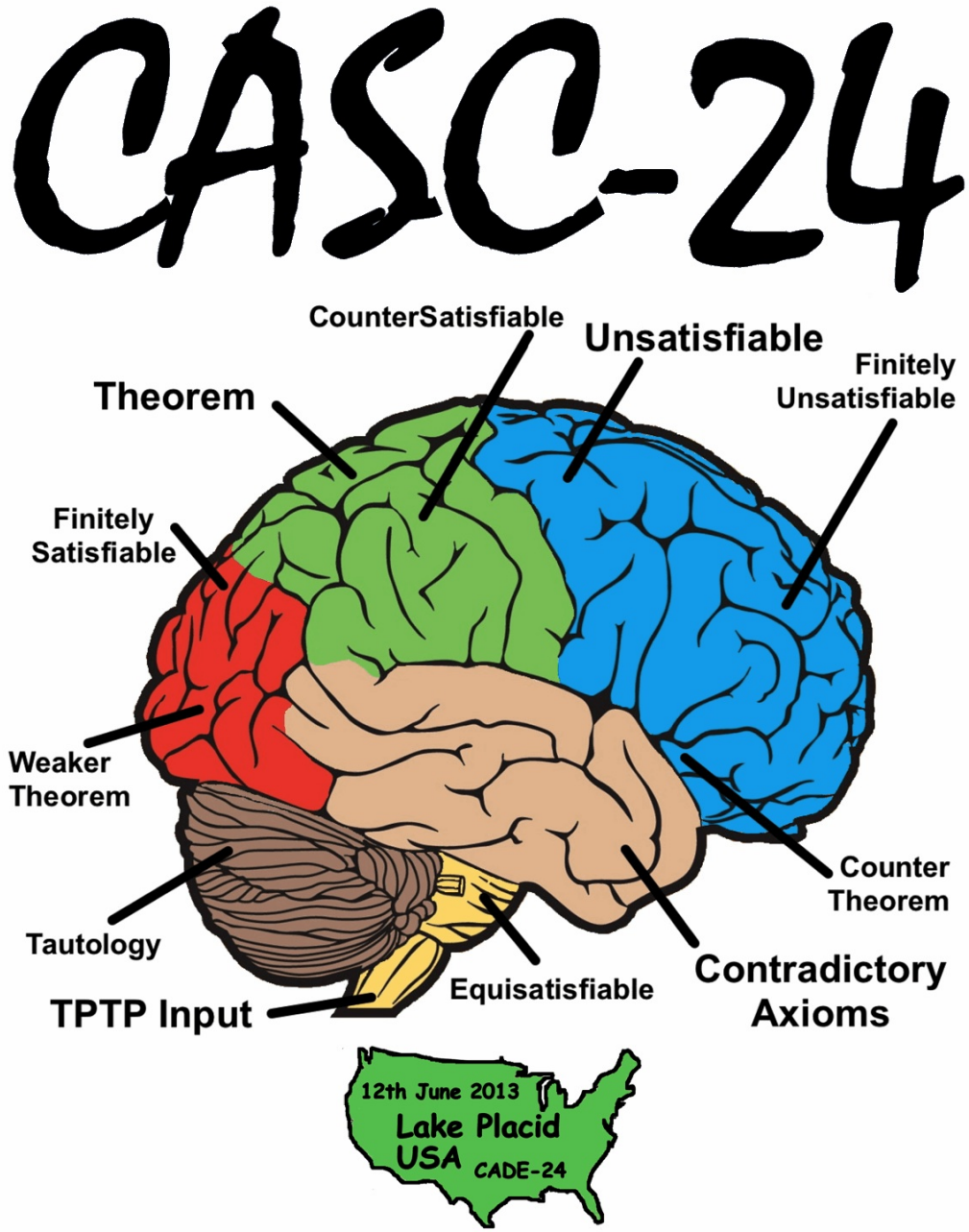
CASC-24
CounterSatisfiable
Unsatisfiable
Finitely Unsatisfiable
Theorem
Finitely Satisfiable
Weaker Theorem
Counter Theorem
Tautology
Equisatisfiable
Contradictory Axioms
TPTP Input
12th June 2013
Lake Placid
USA
CADE-24

CASC 2013

iProver – an instantiation-based reasoner for first-order logic

- based on the Inst-Gen calculus
- modular combination of first-order reasoning with MiniSAT
- redundancy elimination, indexing, ...
- implemented in OCaml
- sort-restricted finite model finding, symmetry reduction

First-order satisfiability (FNT) 150 problems

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prob	122	99	96	79	79	78
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