Tableaux for Relation-Changing Modal Logics

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Dynamic modal logics

- Modal logics are known to describe models.
- Different modal operators:
  - $\Diamond\varphi$, $\Diamond^{-}\varphi$
  - $\mathsf{E}\varphi$
  - $\Diamond_{\geq n}\varphi$
  - $\Diamond^{*}\varphi$
  - ...
- Now, what about operators that can modify models?
  - Change the domain of the model.
  - Change the properties of the elements of the domain while we are evaluating a formula.
  - Evaluate φ after deleting/adding/swapping around an edge.

Why modify models?

- epistemic operators modify models
- $[!\psi]\varphi$: announce that ψ is true, eliminate states of the model where $\neg\psi$ holds (Public Announcement Logic) [Plaza 89].
- In some way these operators are deleting states.
- We will focus on operators that modify the accesibility relation.
- We study various relation-changing operators to help completing the "world map of dynamic modal logics"

Sabotage Modal Logic [van Benthem 2002]

$$\mathcal{M}, w \models \langle gsb \rangle \varphi \text{ iff } \exists \text{ pair } (u, v) \text{ of } \mathcal{M} \text{ such that } \mathcal{M}^-_{\{(u,v)\}}, w \models \varphi,$$

where $\mathcal{M}^-_{\{(u,v)\}}$ is $\mathcal{M}$ without the edge (u, v).

What we know [Löding & Rohde 03]:

- Model checking is PSPACE-complete.
- Satisfiability is undecidable.

Our operators

We have the Basic Modal Logic ($\mathcal{ML}$):

- propositional language + a modal operator $\Diamond$.
- $\Diamond\varphi$: traverse some edge, then evaluate φ.

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Now add new dynamic operators:

	sabotage	bridge	swap
local	$\langle sb \rangle$	$\langle br \rangle$	$\langle sw \rangle$
global	$\langle gsb \rangle$	$\langle gbr \rangle$	$\langle gsw \rangle$

- Local:
  - sabotage and swap: for wv an edge with w the evaluation state, delete or swap it, and move to v
  - bridge: for wv not an edge with w the evaluation state, create it and move to v
- Global: delete, create or swap edges anywhere without moving

Example: local sabotage

- $\overset{\curvearrowright}{\bullet} \rightarrow \circ$ $\qquad \models \quad \langle sb \rangle \langle sb \rangle \top$

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- $\bullet \rightarrow \circ \qquad \models \quad \langle sb \rangle \top$
- $\circ \qquad \bullet \qquad \models \quad \top$

- $\overset{\curvearrowright}{\bullet} \qquad \not\models \quad \langle sb \rangle \langle sb \rangle \top$

Example: local sabotage

- $\overset{\curvearrowright}{\bullet} \rightarrow \circ \quad \models \quad \langle sb \rangle \langle sb \rangle \top$
- $\bullet \rightarrow \circ \quad \models \quad \langle sb \rangle \top$
- $\circ \quad \bullet \quad \models \quad \top$

- $\overset{\curvearrowright}{\bullet} \quad \not\models \quad \langle sb \rangle \langle sb \rangle \top$
- $\bullet \quad \not\models \quad \langle sb \rangle \top$

Example: local swap

- $\bullet \rightarrow \circ$ $\models \langle sw \rangle \langle sw \rangle \top$

Example: local swap

- $\bullet \rightarrow \circ$ $\quad\models\quad \langle sw \rangle \langle sw \rangle \top$
- $\circ \leftarrow \bullet$ $\quad\models\quad \langle sw \rangle \top$

Example: local swap

- $\bullet \rightarrow \circ$ $\models \langle sw \rangle \langle sw \rangle \top$
- $\circ \leftarrow \bullet$ $\models \langle sw \rangle \top$
- $\bullet \rightarrow \circ$ $\models \top$

Tableaux

- Quite common procedure for modal logics in general
- Questions raised by tableaux methods for relation-changing modal logics:
  - what would they look like?
  - will we learn something interesting?

Common features of tableaux for RCML

- We prefix formulas with one constant and a set of edge names (pair of constants)
- $(s, S) : \varphi$
- "φ holds at the state referred to by s in the model variant described by the set of sabotaged/new/swapped edges S".

Common tableau rules

- Boolean rules
- Equational rules
- Clashing rules
- "Unrestricted Blocking" rule: saturate branch with equalities and desiqualities between all pairs of constants:

$$\frac{}{n \doteq m \mid n \dot{\neq} m}\,(ub)$$

- Common notation:

$$\begin{array}{lll} nm \dot{\neq} xy & := & n \dot{\neq} x \;\vee\; m \dot{\neq} y \\ nm \dot{\notin} X & := & \bigwedge_{xy \in X} nm \dot{\neq} xy \end{array}$$

...

Local sabotage tableau

$$\frac{(n,S):\Diamond\varphi}{\begin{array}{c}\dot{R}nm\\ nm\dot{\notin}S\\ (m,S):\varphi\end{array}}\ (\Diamond)$$

$$\frac{\begin{array}{c}(n,S):\Box\varphi\\ \dot{R}nm\\ nm\dot{\notin}S\end{array}}{(m,S):\varphi}\ (\Box)$$

$$\frac{(n,S):\langle sb\rangle\varphi}{\begin{array}{c}\dot{R}nm\\ nm\dot{\notin}S\\ (m,S\cup nm):\varphi\end{array}}\ (\langle sb\rangle)$$

$$\frac{\begin{array}{c}(n,S):[sb]\varphi\\ \dot{R}nm\\ nm\dot{\notin}S\end{array}}{(m,S\cup nm):\varphi}\ ([sb])$$

What we obtained

- Sound and complete calculi for all 6 logics.
- Swap calculus quite complicated (have to maintain swapped and re-swapped edges, quite ugly)

Ending remarks

- Combining incomplete tableaux with model checking provides termination:
  - run tableaux for N steps
  - if closed: UNSAT
  - if open and input formula true in induced model: SAT
  - otherwise: DON'T KNOW
- From the calculi, it is easy to propose equivalence-preserving translations to hybrid logics (modal logics with constants)